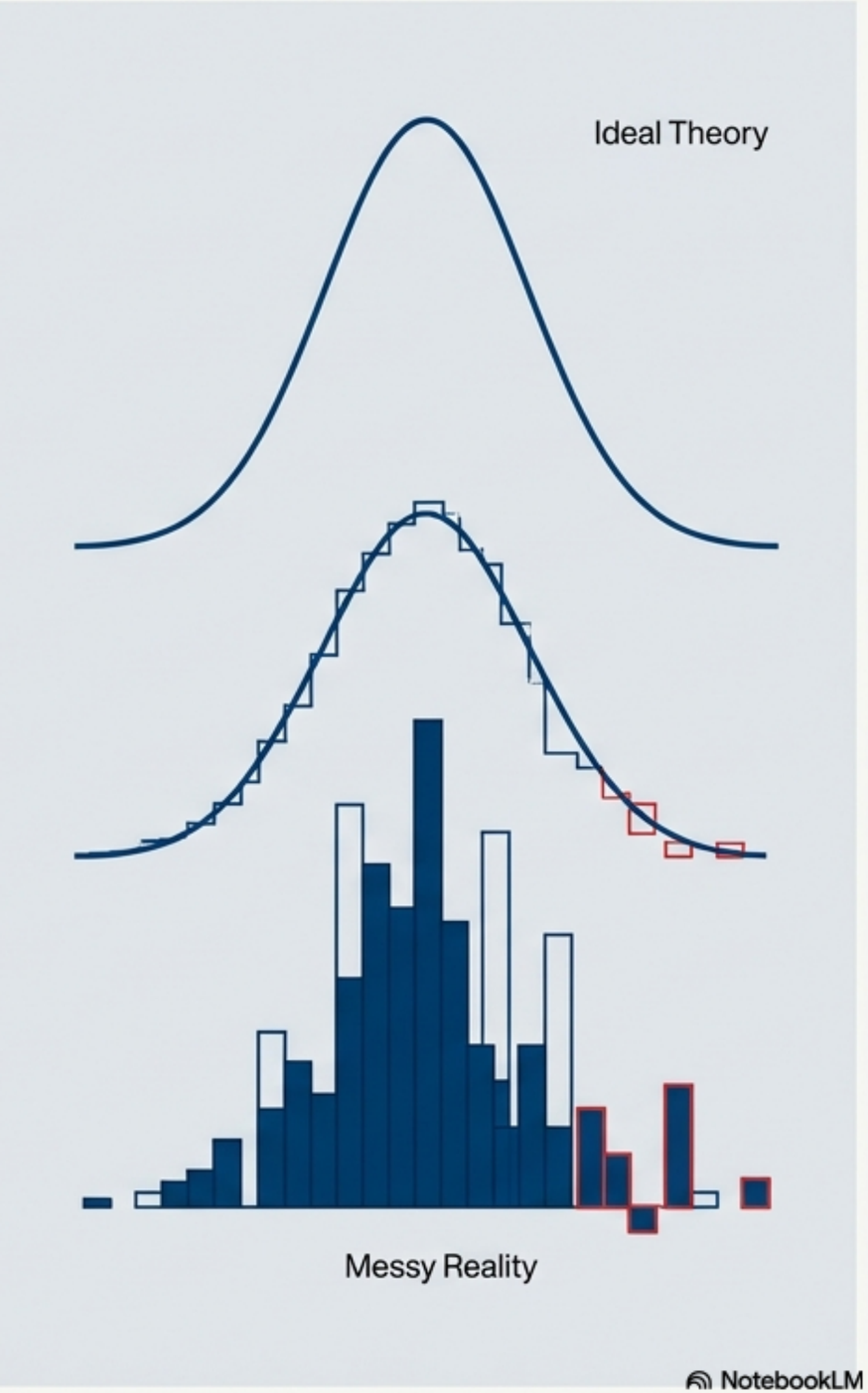
The Normal Distribution in Movement Science

Understanding the bell curve, assessing normality, and handling real-world data

The normal distribution is the mathematical foundation for the statistical tools we use every day, but it is a theoretical ideal rather than a universal law of nature. This guide covers how to leverage the model for prediction, how to diagnose when real movement data breaks the rules, and how to choose the right analytical path when perfection is unattainable.



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The Mathematical Ideal Behind Statistical Inference

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Mean (μ): The center peak of the curve

Standard Deviation (σ): The measure of spread (how wide or narrow)

The normal distribution is a continuous probability distribution defined by two specific parameters: the mean (μ), which determines the center, and the standard deviation (σ), which determines the spread.

This is not merely a “bell-shaped pattern” but a precise function that allows us to connect z-scores to probabilities.

Key Insight

Understanding this model is critical because widely used procedures like t-tests, ANOVA, and regression assume that sampling distributions are approximately normal. Furthermore, this model helps us interpret “error" in measurement and establish normative ranges for athletic performance.

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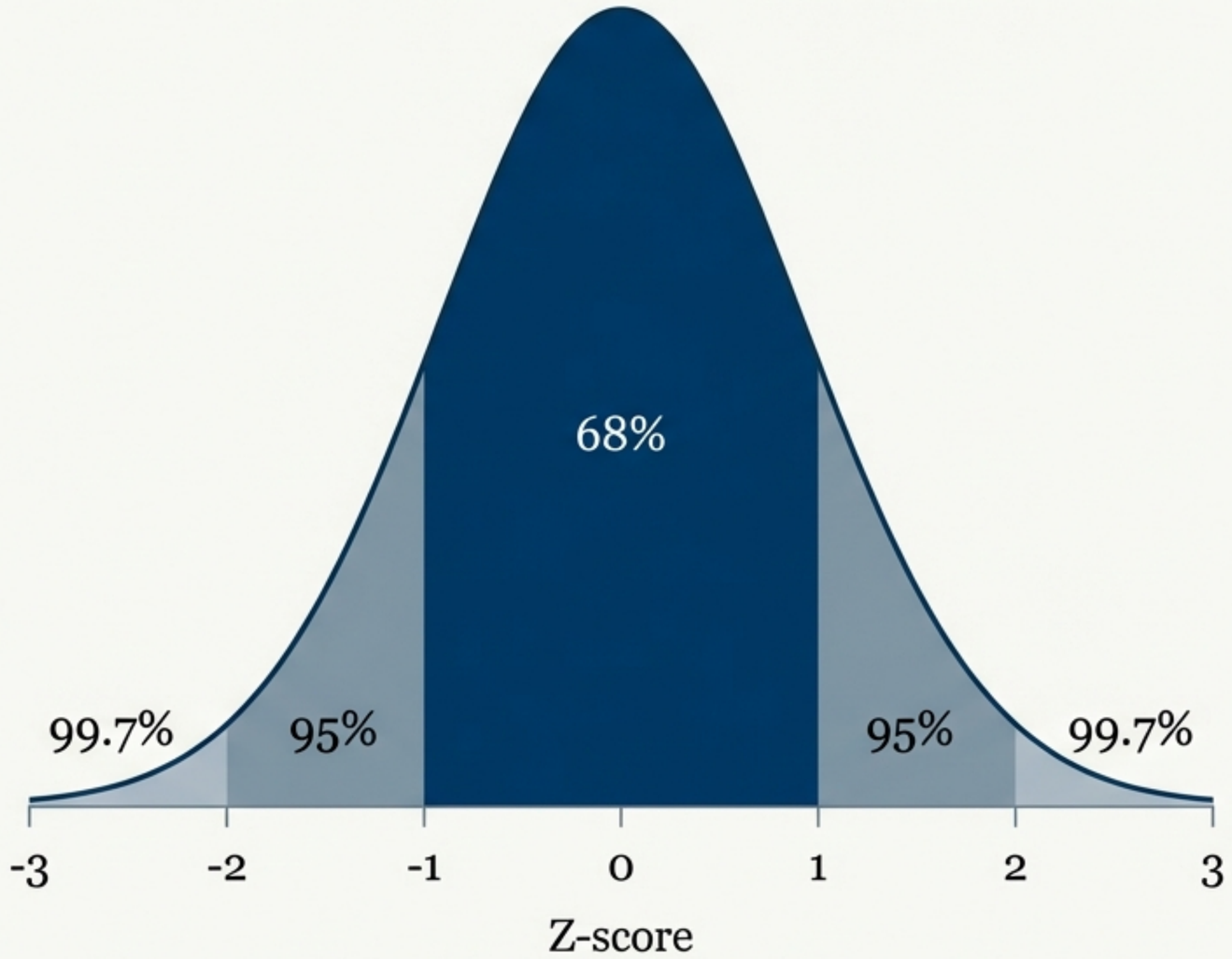
The Rigid Rules of the Perfect Curve

A perfect normal distribution follows strict rules that allow us to predict the frequency of observations.

First, it is perfectly symmetric, meaning the mean, median, and mode are identical.

Second, the tails are asymptotic, extending infinitely without ever touching zero.



The Empirical Rule: Approximately 68% of all observations fall within one standard deviation of the mean (±1σ), 95% fall within two standard deviations, and 99.7% fall within three.

This rule provides an immediate intuition for identifying unusual performance values.

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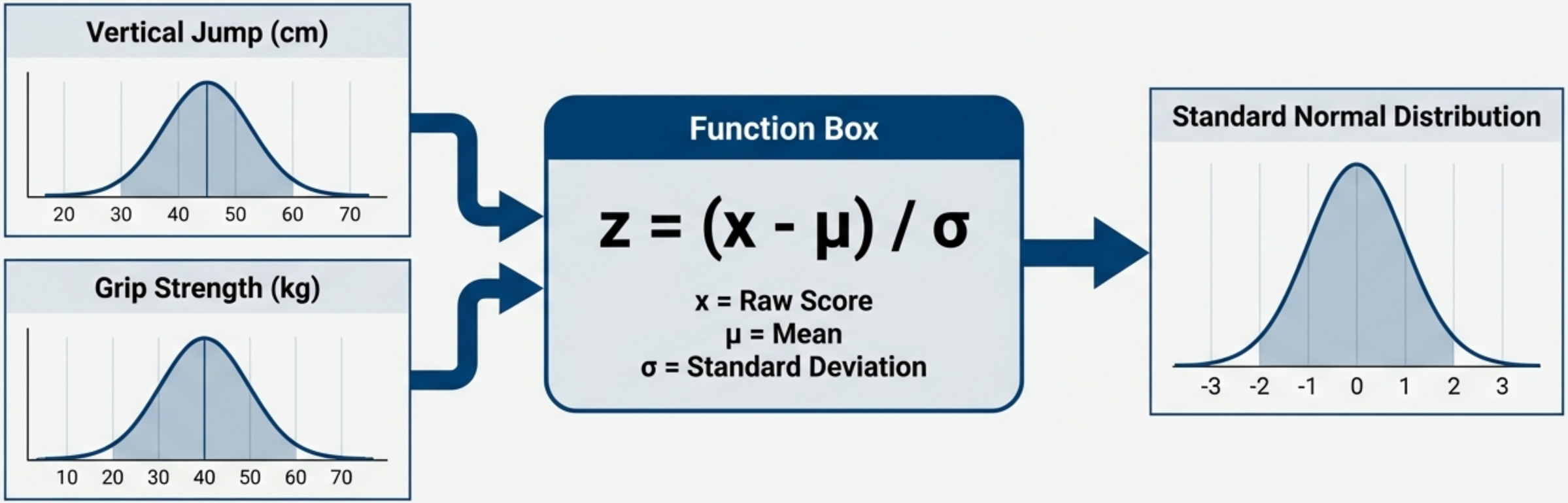
The Universal Translator: Standardizing Data


Vertical Jump (cm)
20 30 40 50 60 70
Grip Strength (kg)
10 20 30 40 50 60 70
Function Box
z = (x - μ) / σ
x = Raw Score
μ = Mean
σ = Standard Deviation
Standard Normal Distribution
-3 -2 -1 0 1 2 3

We can compare completely different variables—like vertical jump in centimeters and grip strength in kilograms—by converting them into a single standard language: the z-score. The standard normal distribution is a special case where the mean is 0 and the standard deviation is 1.

By applying the formula, we strip away the original units. A z-score of +1.5 represents the same relative level of performance whether we are measuring reaction time in milliseconds or force in Newtons. This standardization is the mechanism that makes universal probability tables possible.

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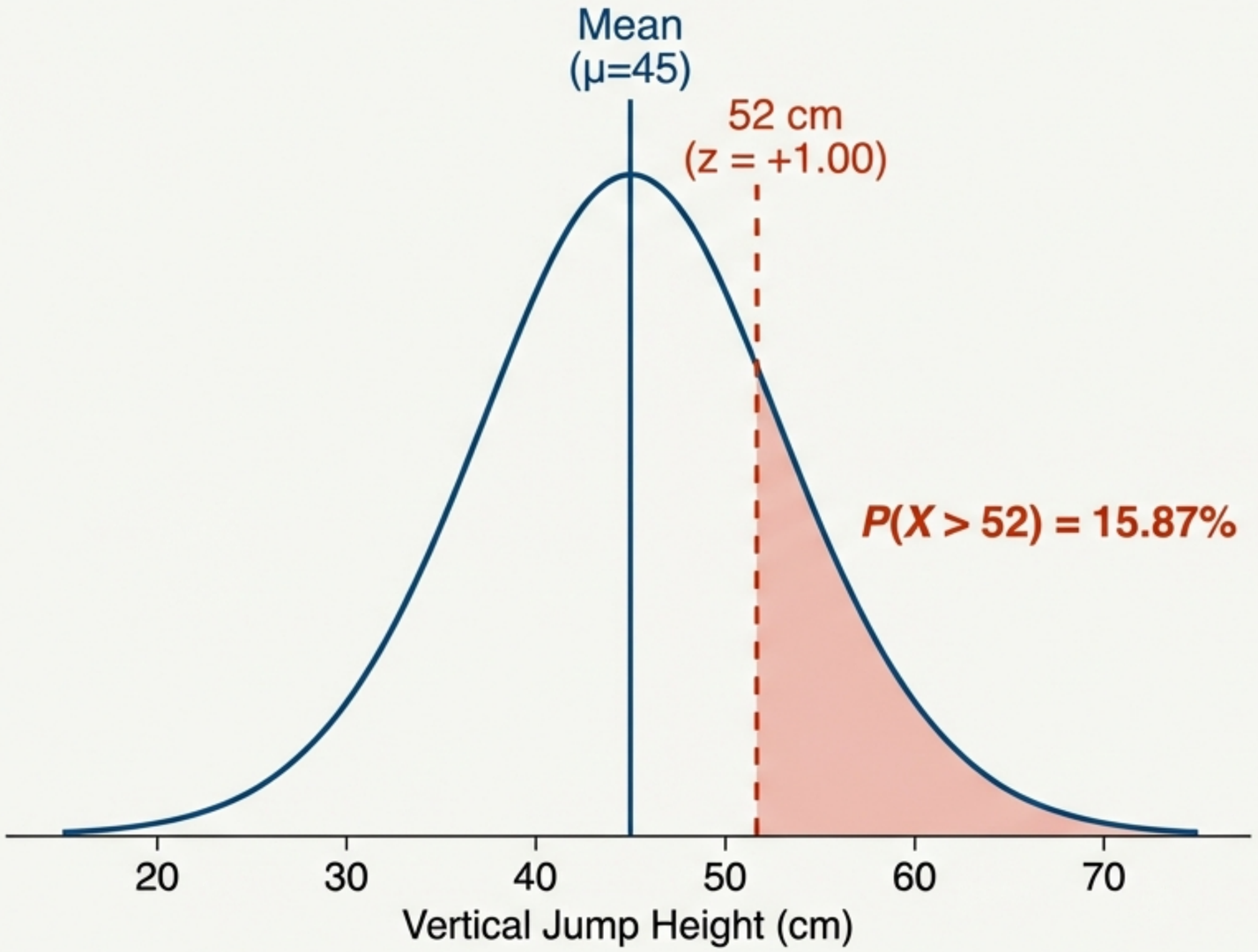
Calculating Probability: The Jump Height Example



Scenario: Collegiate athletes with vertical jump $\mu = 45$ cm and $\sigma = 7$ cm.

Question: What is the probability of jumping higher than 52 cm?

Calculation: First, calculate the z-score: $(52 - 45) / 7 = 1.00$.

Result: Using the z-table, 84.13% of the distribution falls below this point. Consequently, there is a 15.87% probability of observing a jump higher than 52 cm.

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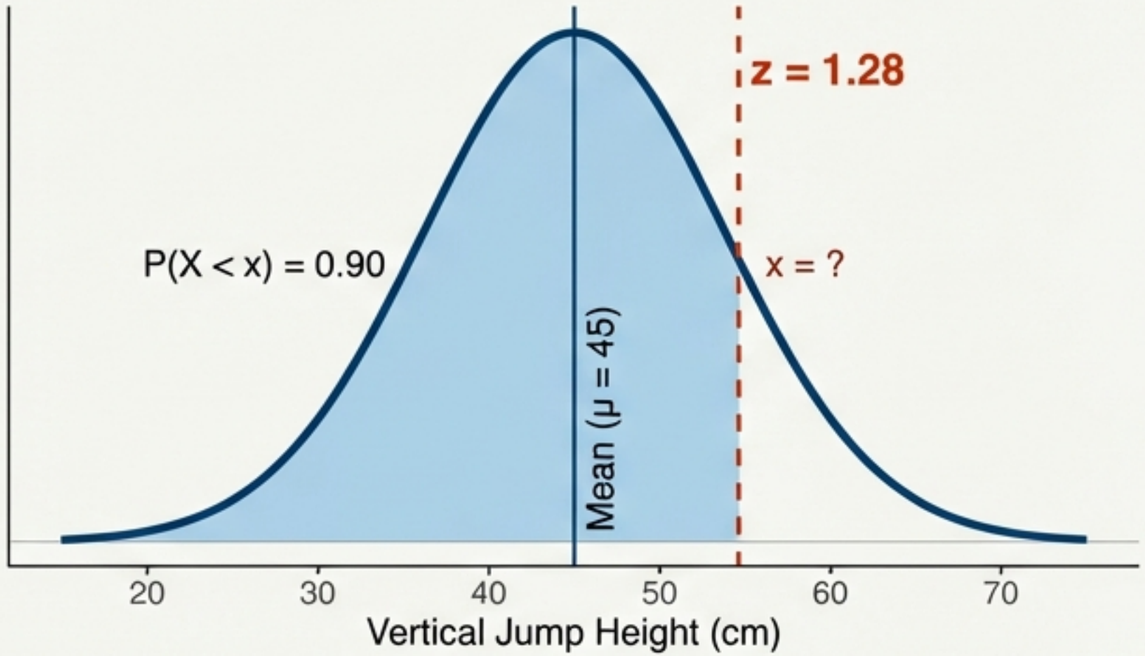
Working Backward: Establishing Performance Cutoffs

Finding the raw score associated with a specific percentile rank.

We can also reverse the process to determine the performance score required to reach a specific percentile rank. Using the same jump data (μ =45, σ =7), let us determine the height of the 90th percentile.

The Calculation: A cumulative probability of 0.90 corresponds to a z-score of approximately 1.28. We convert this back to raw units using the formula $x = \mu + (z \times \sigma)$.

Result: 45 + (1.28 × 7) = 53.96 cm. Therefore, a jump of approximately 54 cm is the cutoff that separates the top 10% of athletes from the rest.



$$x = \mu + (z \times \sigma)$$
$$x = 45 + (1.28 \times 7)$$
$$x = 53.96 \text{ cm}$$

Figure 3: Normal distribution demonstrating the calculation of the 90th percentile raw score based on the jump height parameters.

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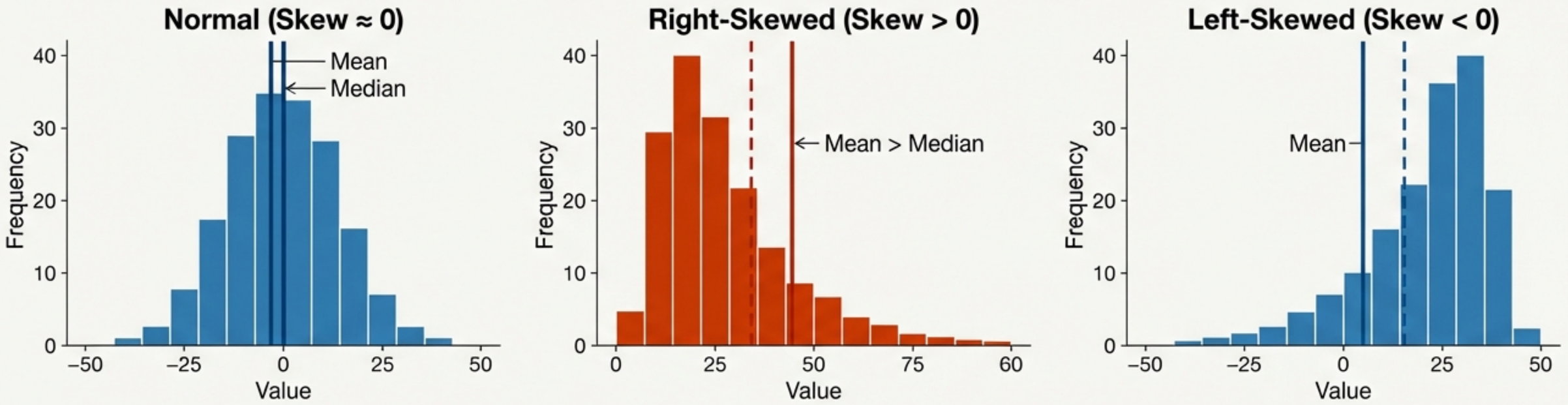
Reality Check: The Asymmetry of Skewness



Real movement data rarely fits the perfect curve; biological limits often stretch the distribution to one side. Skewness quantifies this asymmetry.

Movement Science Context: Reaction time data typically exhibits Positive (Right) Skew. This occurs because there is a physiological floor (we cannot react instantly, bounding the left side), but lapses in attention can create very long response times, dragging the tail to the right. In these cases, the mean is pulled toward the tail and no longer represents the typical athlete as accurately as the median.

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The Weight of the Tails: Understanding Kurtosis

Defining Kurtosis: More Than Peakedness

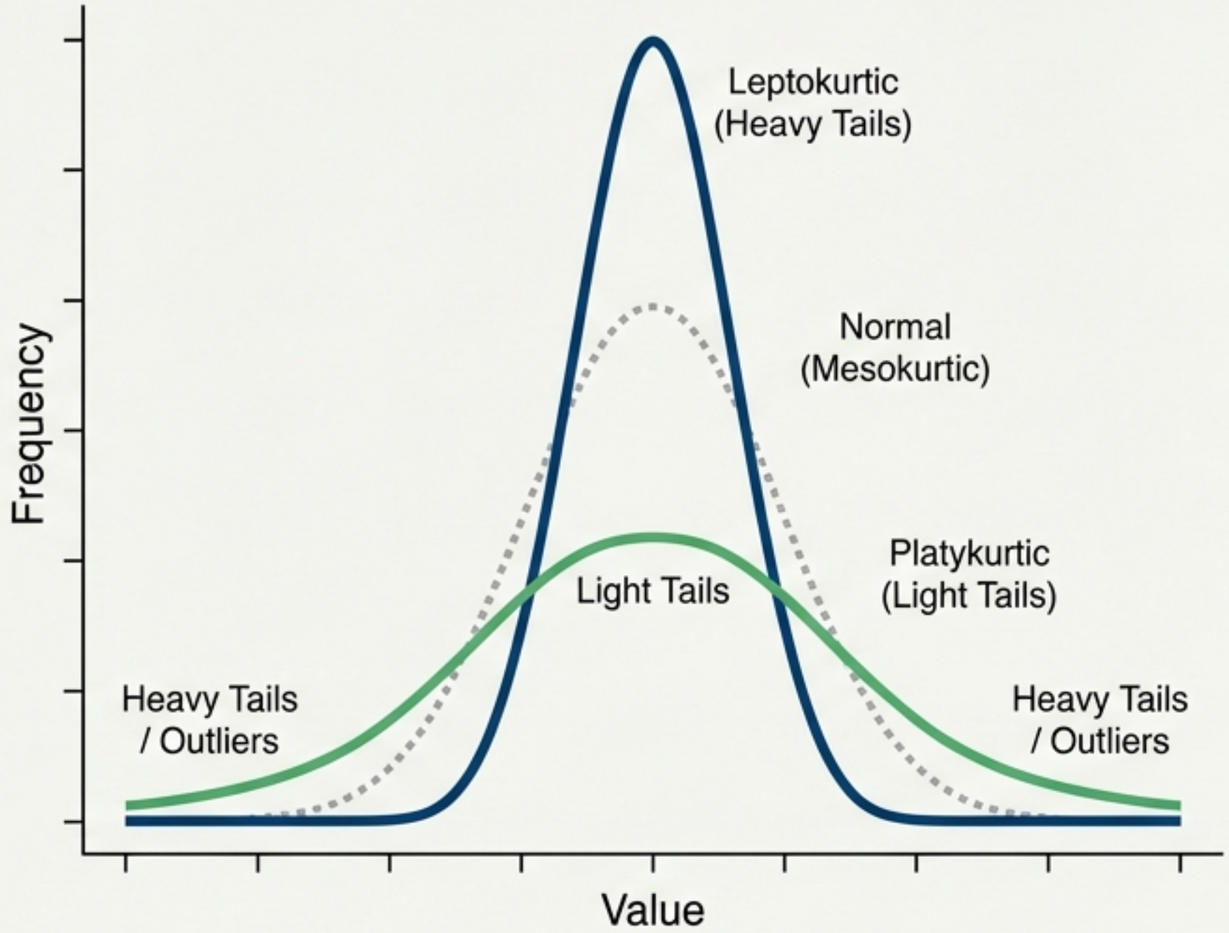
Kurtosis reveals the “extremity” of the data. While historically described as “peakedness,” it is more accurately interpreted as a measure of tail weight.

Leptokurtic (Heavy Tails)

A distribution with positive kurtosis produces more extreme outliers than the normal model predicts. This is common in strength data where occasional supreme efforts or measurement artifacts create “heavy tails.”

Platykurtic (Light Tails)

Negative kurtosis indicates fewer extremes, often seen when data is truncated or when disparate groups are combined, flattening the distribution.



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Trust Your Eyes: Visual Assessment Tools

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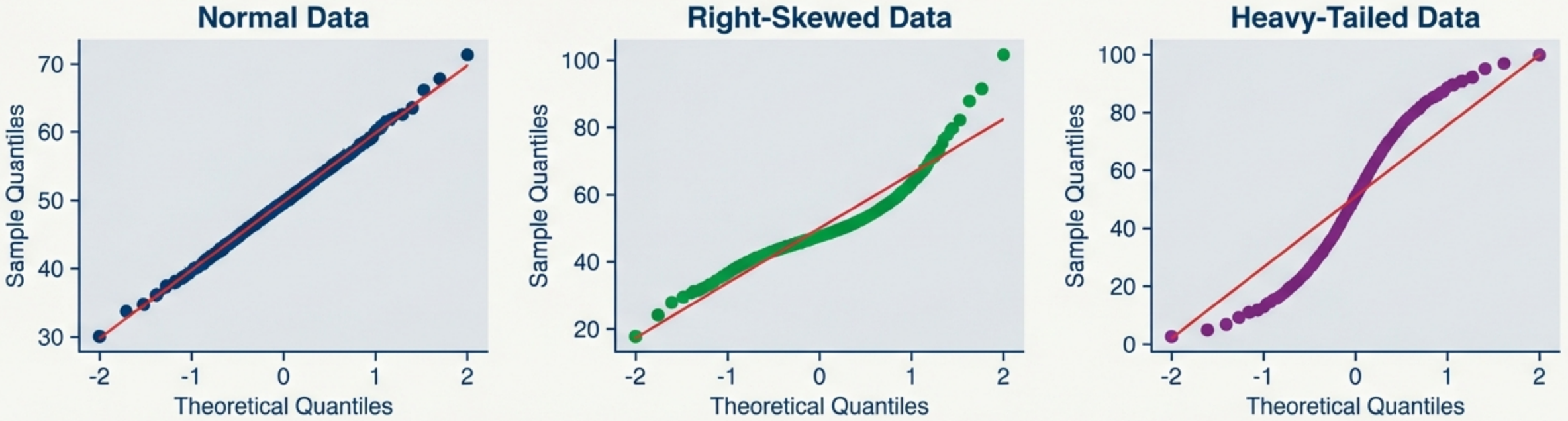
The most effective way to judge normality is not a single calculation, but a visual inspection of the data's shape. While histograms provide a general overview, the Q-Q Plot (Quantile-Quantile) is the superior diagnostic tool.

Interpretation: In a Q-Q plot, data points that hug the straight diagonal line indicate normality. Systematic deviations reveal the specific nature of the violation: an "S-shape" indicates skewness, while points curving away from the line at the ends indicate heavy or light tails.



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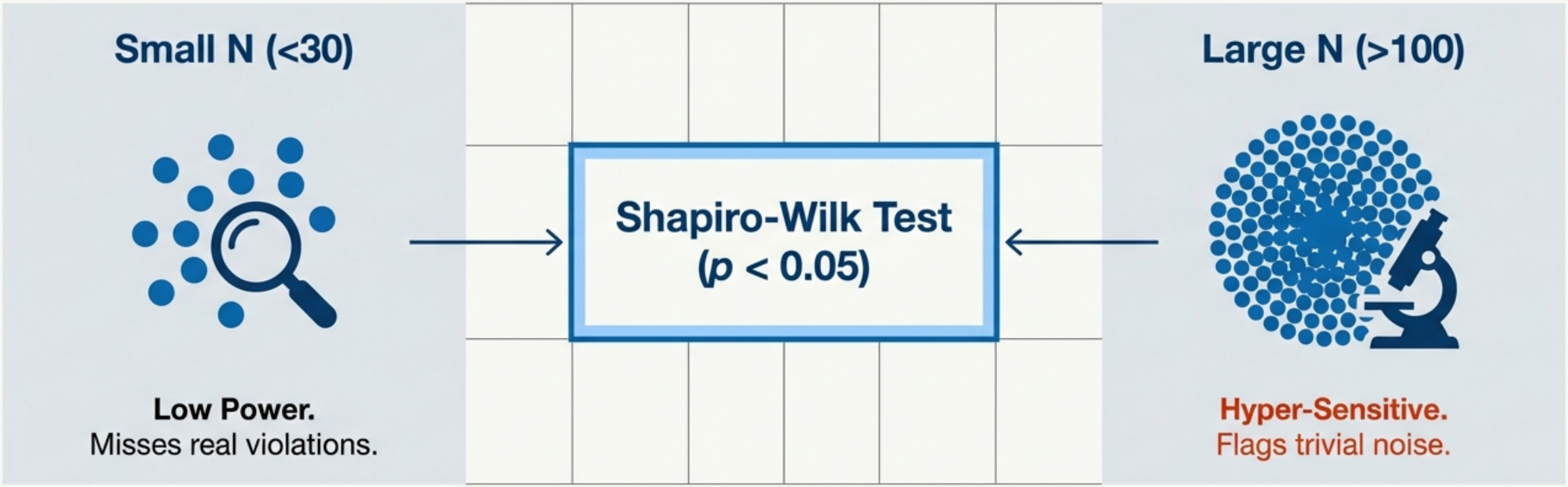
The Paradox of Formal Normality Tests

Formal tests like the Shapiro-Wilk provide a p-value for normality (H_0: data is normal), but they suffer from a "Sample Size Paradox" that requires careful interpretation.



The Paradox: In small samples ($N < 30$), these tests have low power and often miss real violations. Conversely, in large samples ($N > 100$), they become hyper-sensitive, flagging trivial, irrelevant deviations as statistically significant.

A p-value < 0.05 does not always mean your analysis is doomed; it simply means the data is not perfectly normal.

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The Convergence Rule: Making a Decision

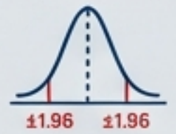
Because no single method is perfect, we must integrate visual evidence with formal test results based on our sample size.

The Strategy: With small samples, formal tests often fail to warn us of danger, so we must rely on our eyes. With large samples, formal tests often cry wolf over tiny imperfections, so again, we trust the visuals. It is only in the middle ground that the two methods are balanced partners.

	Sample Size Category		Recommended Action
1	Small Samples ($N < 30$)	→	Trust Visuals (Test Underpowered)
2	Moderate Samples ($30 - 100$)	→	Use Both (Visuals + Test)
3	Large Samples ($N > 100$)	→	Trust Visuals (Test Over-sensitive)

Quantitative Check

Quantitative Check: Z-skew and Z-kurtosis values exceeding **± 1.96** indicate significant departures.



Decision Table for Conflicting Evidence

	Visual Assessment	Formal Test Result	Sample Size	Recommended Action
1	Approximately normal	$p < 0.05$	$n < 30$	Use parametric methods; test likely underpowered
2	Visual shows normality	Test rejects ($p < 0.05$)	$n > 100$	Trust visual assessment; proceed with parametric methods
3	Visual shows departure	Test does not reject ($p > 0.05$)	$n < 30$	Trust visual assessment; use transformation, robust methods, or nonparametric approaches
4	Borderline/ambiguous	Any Result	Moderate (30-100)	Use robust methods or report parallel analyses

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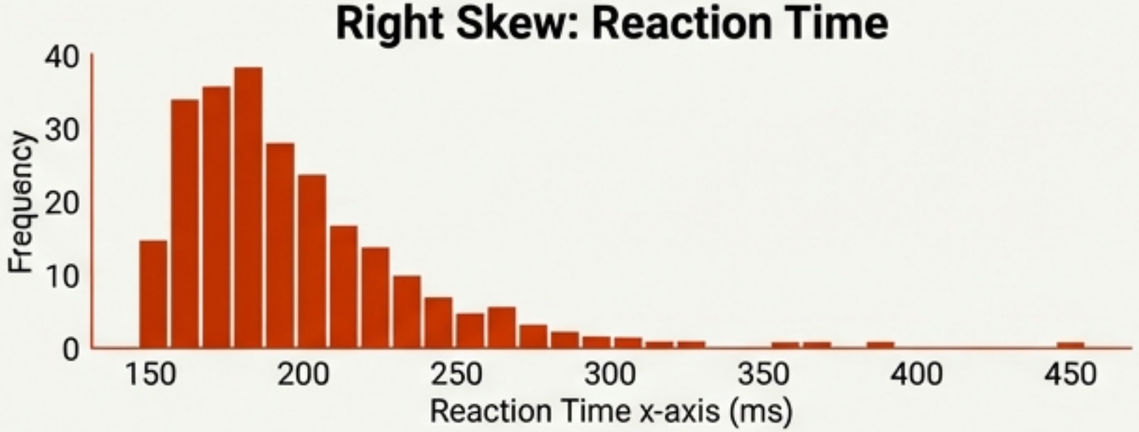
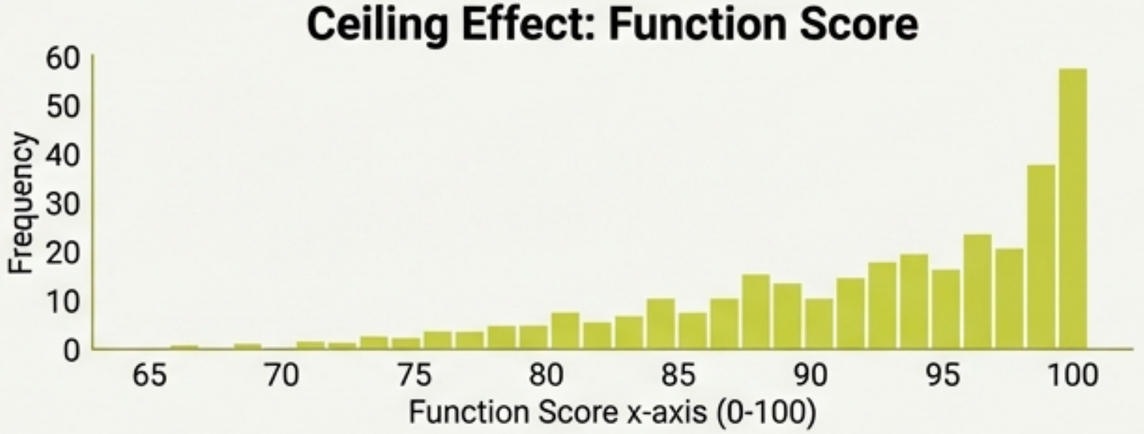
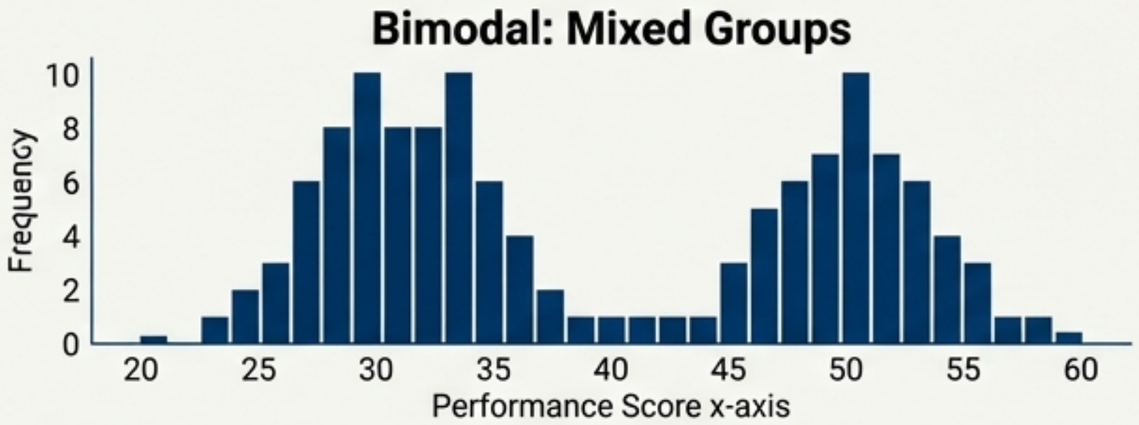
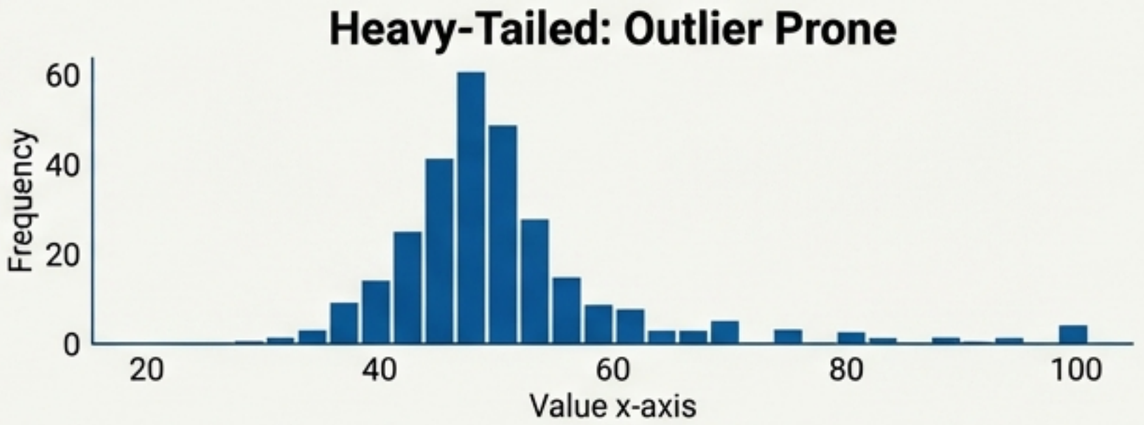
Common Non-Normal Patterns in Movement Science









Certain types of movement data systematically violate normality, and recognizing these signatures is crucial.

- Right Skew: Seen in reaction times and EMG amplitude (bounded by zero).
- Ceiling/Floor Effects: Seen in pain scales or balance tests (data truncated).
- Bimodal: Mixed populations (e.g., trained vs. untrained athletes).
- Heavy Tails: Often caused by equipment error or motivational outliers.

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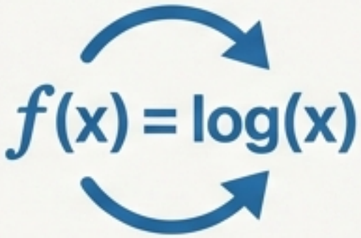
Remediation: Handling Non-Normal Data

When data violates the normal assumption, we have three main options to ensure our analysis remains valid.

Transform



Transform

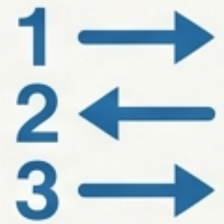
Apply mathematical adjustments like Log-transformation to compress long tails (ideal for Reaction Time).

Robust Methods

Robust Methods

Use outlier-resistant statistics (Median/IQR) or robust inference (Welch's t-test, Bootstrapping).

Non-Parametric



Non-Parametric

Switch to rank-based tests like Mann-Whitney U if violations are severe.

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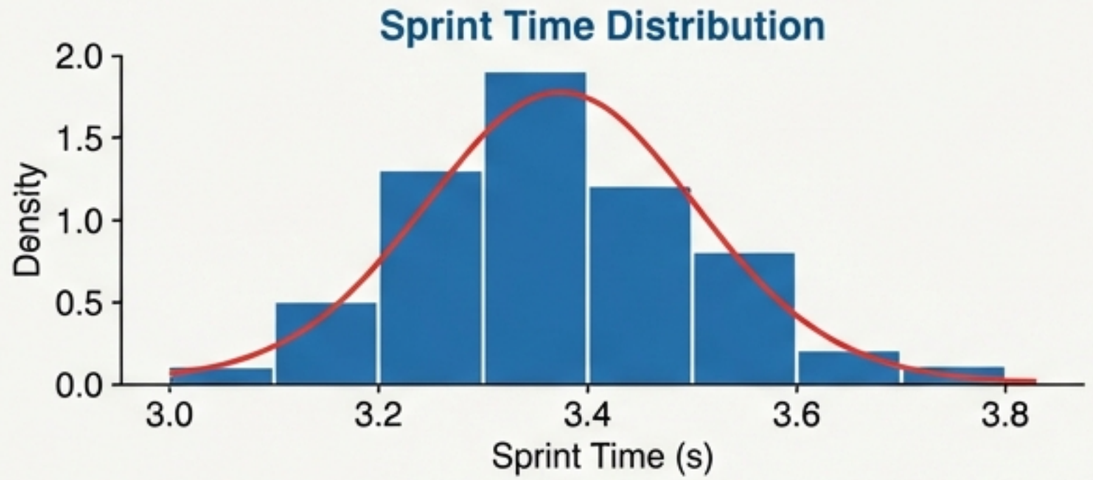
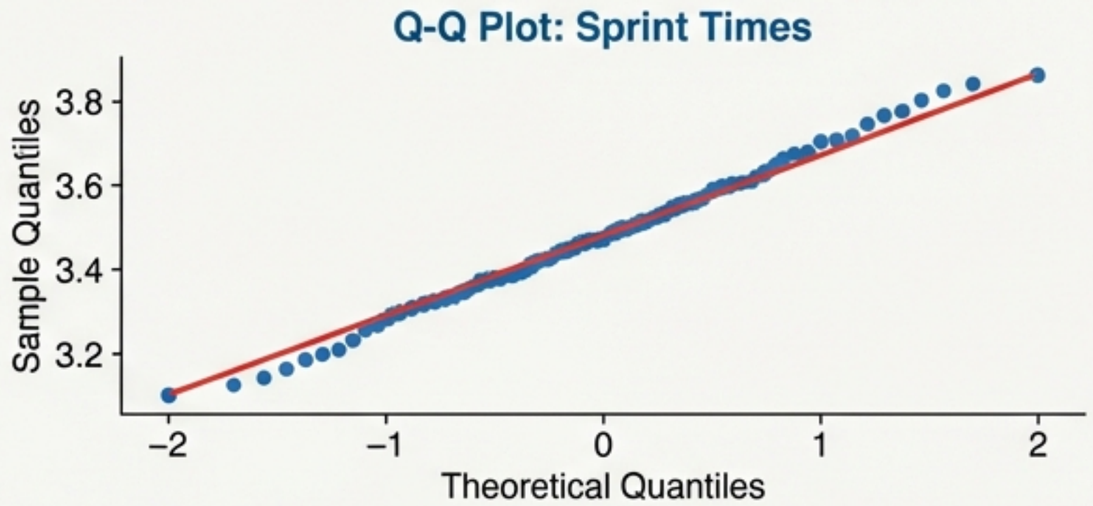
Applied Workflow: Assessing Sprint Times

The Visual Evidence





The Analytical Narrative

Let us apply this workflow to a dataset of 20-meter sprint times ($N = 40$).

Step 1 (Visual): The histogram shows reasonable symmetry, and the Q-Q plot points hug the diagonal line with only minor waviness.

Step 2 (Metrics): Skewness is 0.15 and Kurtosis is -0.22. Both are negligible.

Step 3 (Test): The Shapiro-Wilk test yields $p = 0.524$, meaning we fail to reject normality.

Decision: We proceed with a parametric t-test. The data is approximately normal, and the sample size provides additional robustness.

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Conclusion: Context Over Rules

> “Normality is not a binary state of “true” or “false,” but a continuum where context determines our analytical path.”

The Goal

The goal is not to worship the bell curve but to understand when it is a useful model.

The Method

Always prioritize “Trust Your Eyes” (visual assessment) over blind reliance on p-values.

The Application

Recognizing the nature of your data ensures that your statistical conclusions reflect the reality of human movement.

Summary Points: Understanding, Assessment, and Application.

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